feature article

Anderson localization of ultracold atoms

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To study localized matter waves, two experimental groups hold a Bose–Einstein condensate in the grip of a disordered but tunable optical potential formed by interfering laser beams.

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In 1958 Philip Anderson published a paper suggesting that certain materials can suffer a sudden phase transition, from conductor to insulator, under a slight change in the amount of disorder in the material. At the time, every physicist would have understood that disorder hinders electron mobility and thus decreases the material's conductivity. But until then, no one had predicted its complete cancellation past a certain amount of disorder. Even 20 years after his seminal article, when Anderson received the Nobel Prize in Physics in 1977, the effect was still far from fully understood. (For a historical account, see the article by Ad Lagendijk, Bart van Tiggelen, and Diederik Wiersma on page 24.) Today, questions still remain—among them, what is the exact critical value of the disorder at the transition? and how do interactions change the picture? Fortunately, researchers are now in a position to answer some of those questions. One approach is to observe the behavior of ultracold atoms in disordered optical potentials—the focus of this article.

Matter waves that go nowhere

Anderson's initial model starts from the tight-binding model of an electron in a crystal,[1] considered as a periodic lattice of potential wells, as pictured in figure 1a. The electron can hop from one site to nearest-neighboring ones by quantum tunneling. For large enough tunneling amplitudes, one obtains as a stationary-state solution of the Schrödinger equation an extended wavefunction that describes an electron able to freely propagate through the crystal. That propagation in a perfect crystal can also be described as a Bloch wave in a conduction band.[1] If disorder is added to the tight-binding model by randomly shifting the energies of the various trapping sites, with a distribution of width W, propagation is hindered, a situation that corresponds to ohmic conduction.

Anderson's conjecture, based on a general mathematical argument, is the existence of a level of disorder beyond which the conductivity is more than reduced, it is totally cancelled. The breakdown of conduction is associated with a sudden change in the electronic wavefunction, which ceases to be extended and becomes spatially localized with exponentially decreasing tails. The surprise is that localization happens even if the electron remains able to tunnel between neighbor lattice sites: The various quantum amplitudes associated with all those paths cancel when they are added.

Indeed, Anderson localization (AL) is, in essence, an inter-

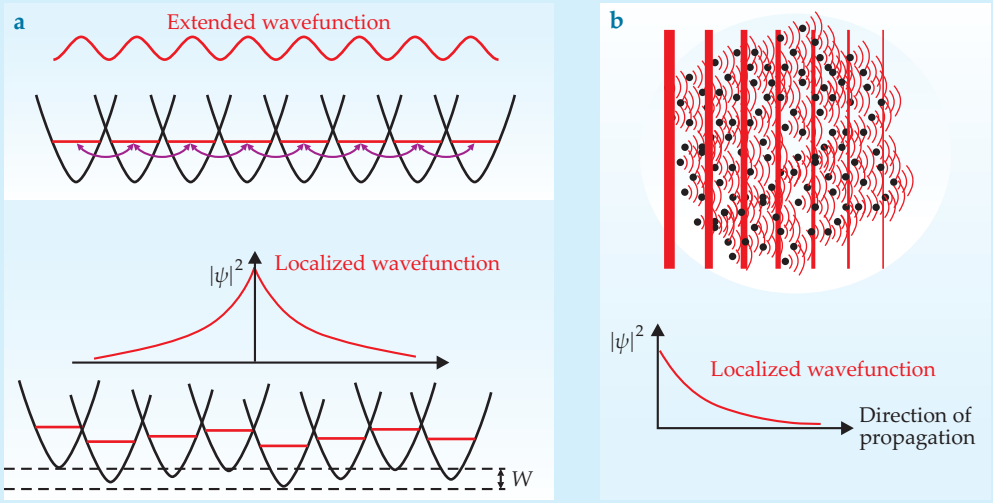


Figure 1. Hopping electrons or scattered waves. **(a)** If electrons are allowed to tunnel between neighboring sites of an ordered lattice, one obtains a freely propagating wave as the solution of the Schrödinger equation. If the regularity of the lattice is broken by randomly changing the depth of the potential at each lattice site, the solution may become localized—that is, the wavefunction ψ decays exponentially in space (the horizontal axis). The decay, a signature of Anderson localization, occurs when the level W of disorder is large enough. **(b)** When a plane wave (the vertical lines show the wavefronts) tries to propagate in a medium with many scatterers, the rescattered wavelets interfere in the forward direction and add coherently to the initial wave. If the amount of scattering is large enough, the forward interference is destructive and the amplitude of the wave decreases exponentially along the propagation direction—the wave is localized.

ference phenomenon. And after Anderson's conjecture based on the tight-binding model of hopping electrons, an equivalent model based on wave physics emerged.[2] The basis of that approach is the equivalence between the running Bloch wave describing a particle freely propagating in a conduction band of a perfect crystal and a matter wave freely propagating in a homogeneous nonabsorbing medium.[3] Adding disorder to the latter model can be done by introducing impurities, from which the matter waves scatter and propagate diffusively through the crystal.

The wave model of ohmic conduction is based on the assumption that diffusion is incoherent and that one can add the intensities of the wavelets scattered from impurities. In contrast, when one adds the *amplitudes* of the scattered waves, as should be done according to basic quantum laws, destructive interference may cancel the propagation in the forward direction or in any other direction (figure 1b). That occurs when the mean free path ℓ between two scattering events is smaller than the wavelength λ of the wave divided by 2π—that is, when the dephasing between successive scatterings is less than about 1 radian. The condition, known as the Ioffe–Regel criterion, was introduced in the context of AL by Nevill Mott.[3] It stresses the existence of a threshold, or mobility edge, at which the transition from extended to localized happens.

Despite its wide acceptance, the exact status of the Ioffe–Regel criterion is still not completely clear. First, rigorous derivations are based on the scaling theory developed by the famous "gang of four" (Elihu Abrahams, Anderson, Donald Licciardello, and T. V. Ramakrishnan), which does not yield the precise value of the ratio ℓ/λ associated with the transition. Second, the situations in dimensions lower than three have many intriguing features. For instance, a general result of scaling theory is that in one dimension all states are localized, and there is no a priori mobility edge to separate localized states from extended ones. But as we will see below, the situation may be more subtle.

Ultracold atoms

Although Anderson was thinking of electrons in his 1958 paper, directly observing AL of electrons turns out to be difficult. A number of phenomena can mask the single-particle quantum effects induced by disorder: Electrons repel each other and are affected by lattice vibrations, a kind of disorder that is not static and cannot produce AL. Moreover, directly observing electron wavefunctions in solids is quite challenging; most evidence of AL is indirect and based on conductivity measurements.

In contrast, ultracold atoms, which are also genuine quantum particles, allow one to address the core of the phenomenon that Anderson predicted, since interactions can be reduced to a negligible level. Moreover, single-atom matter waves can be directly visualized by absorption or fluorescence imaging of the atomic density of a dilute Bose–Einstein condensate (BEC), as outlined in the box above. Indeed, such an ideal quantum gas is nothing but many independent atoms in the same one-atom wavefunction.

One can then experimentally monitor the behavior of an atomic wavefunction placed in a disordered potential.[4] That's possible with an appropriately designed light pattern, since the atoms can be submitted to an optical potential directly proportional to laser light intensity for a laser wavelength far enough from a resonance. That potential is due to the coupling of the laser's electric field with the induced atomic electric dipole, and it results in an interaction energy proportional to the squared electric field, averaged over time.[5]

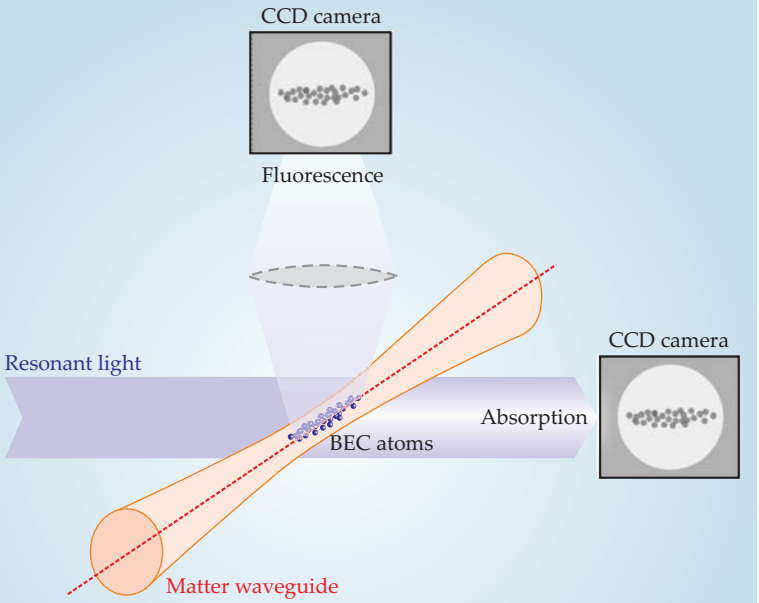
To apply a disordered light intensity to the atoms, one could image a computer-generated disordered mask.[6] Instead, we directly generate the disordered optical potential by the interference of suitably arranged laser waves. Indeed, we have reproduced two different situations that correspond to the two different approaches presented in figure 1. One of us (Aspect) and colleagues at the Institut d'Optique in Palaiseau, France, generated a laser speckle pattern, producing a perfectly controlled disordered potential for ultracold atoms.[7,8] The other (Inguscio) and colleagues at LENS (European Laboratory for Nonlinear Spectroscopy) in Florence, Italy, created "certain random lattices" by adding one optical lattice to another having a noncommensurate period.[9,10] In both cases, ultracold atoms from a noninteracting BEC were then loaded in the optical potential to directly observe AL.

Laser speckle

To experience laser speckle, one needs only to look at the bright and dark spots of light scattered when a helium–neon laser shines on a rough surface. A similar, random light-intensity pattern is formed for the atoms by passing a laser beam through a ground-glass diffusing plate. Each point of the speckle pattern receives many wavelets issued from different points of the plate; the wavelets' interference gives rise to the pattern. Laser speckle is a well-characterized random process.[11] The light-intensity distribution in the speckle

Imaging a single-atom wavefunction with a Bose–Einstein condensate

A noninteracting Bose–Einstein condensate is composed of many atoms described by the same wavefunction. When a resonant laser beam is passed through such a cloud trapped in a waveguide, the absorption is proportional to the atomic density—that is, the squared modulus of the wavefunction integrated along the line of sight. The fluorescent light scattered by the atoms also yields a faithful image of the atomic density.



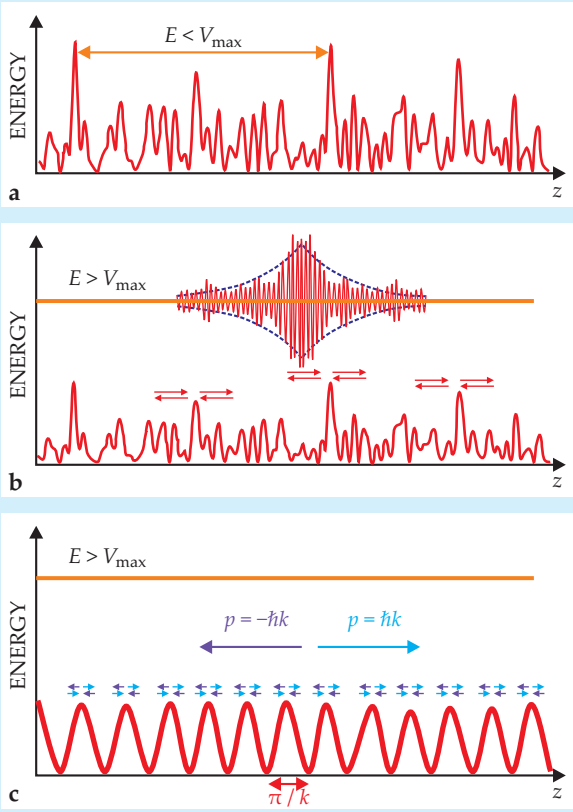


Figure 2. Localization of a particle in a speckle potential. **(a)** When the particle's energy E (orange) is smaller than the highest peaks V_{max} of the potential (red), localization results from trapping between the two peaks and can be understood in classical terms. **(b)** If the energy is larger than the highest peak, solving the Schrödinger equation one can find an exponentially localized wavefunction (the purple envelope), which results from the addition of many wavelets scattered on the peaks, as suggested by the arrows. This quantum localization phenomenon is akin to Anderson localization. **(c)** In a periodic potential of spatial frequency $2k$, a particle with momentum $p = \hbar k$ cannot propagate because it is Bragg reflected, the result of many wavelets scattered from the periodic structure. Localization in panel b can thus be interpreted as Bragg reflection of each momentum component of the particle by the corresponding spatial frequency component of the disordered potential.

pattern is a decaying exponential, from which we can estimate the statistics of peak heights. The autocorrelation function of the intensity pattern, which characterizes the speckle grains' size and shape, is controlled by the shape of the illuminated surface on the scattering plate.

In the Institut d'Optique experiment, the typical size of a grain along the z-axis can be as small as a micron; the transverse size is 50 times as large.[7] The atoms are guided along the z direction by a strong and narrow laser beam, which acts as a matter waveguide with a typical diameter of only a few microns. The atoms are thus transversely confined but they can freely move along the z direction. When the laser speckle pattern is applied, the guided atoms experience a transversely invariant but longitudinally disordered potential that affects their motion along the z direction. Thus we have a situation allowing us to study one-dimensional AL.

What's the point of studying 1D AL? According to scaling theory, whatever the (nontrivial) potential, for each energy E of a particle of mass M there exists a solution of the Schrödinger equation that describes a localized state. However, when we considered the behavior of ultracold atoms placed in a 1D optical speckle, we found unexpected properties of 1D AL,[8,12] as we now explain.

Many theoretical studies of AL of waves are based on a model of scattering impurities described as randomly positioned Dirac peaks—infinitely high and narrow potentials. But a laser speckle potential $V(z)$, as drawn in figure 2, is made of randomly positioned peaks of finite height and finite width—a very different situation. The peak-height distribution is a decaying exponential for one thing, which means that it is exponentially improbable to find peaks with a height many times the average value of the potential and that in a finite sample the potential has a maximum value, V_{max}. Moreover, the random potential has no spatial variation more rapid than the typical size σ_s of a speckle grain. The two features entail dramatic consequences.

Consider the first feature. In a sample of finite size, no speckle peak has a height greater than some maximum value V_{max} (figure 2a). We can then distinguish two regimes. If we take a particle whose energy is significantly below V_{max}, we're likely to find two peaks larger than the particle energy and thus a trivially localized state. That's the equivalent of classically trapping a particle between two potential barriers.[13] But what happens if the particle has an energy larger than V_{max} (figure 2b)? A classical particle would propagate from one end of the sample to the other without being blocked. A numerical solution of the Schrödinger equation in such weak disorder shows that the wavefunction may be exponentially localized for well-chosen parameters. One can interpret that localization's origin in the interference between the many wavelets scattered from the speckle potential.

To better understand the localization, think of the disordered potential as the superposition of many Fourier components, and think of the atom as a matter wave interacting separately with each of those components. If there is a nonvanishing component at spatial frequency $2k$, the atom with momentum $p = \hbar k$ is Bragg reflected (figure 2c). Furthermore, in a speckle potential no Fourier component with a spatial frequency larger than $2/\sigma_s$ exists. So, according to this simple reasoning, there is a cutoff value $k_{co} = 1/\sigma_s$ such that a matter wave with a momentum larger than $p_{co} = \hbar k_{co}$ will not be localized. We thus have an effective mobility edge, and atoms in a 1D speckle potential are expected to have a behavior akin to AL.

Watching the wavefunction

Reaching the regime in which we expect to see AL of ultracold atoms in a 1D speckle disorder demands apparently contradictory conditions. On the one hand, the energy of atoms must be larger than the height of the largest speckle peak in the sample to avoid classical trapping. On the other hand, the energy must be lower than the energy $p_{co}^2/2M$, which corresponds to the effective mobility edge. Those two conditions are compatible if the speckle's grain size is small enough. And thanks to wide-aperture optics, it's on the order of the laser's wavelength.

It's then possible, by releasing a BEC in a 1D optical potential, to follow what happens to an ensemble of noninteracting atoms, all in the same single-atom wavefunction, with momenta smaller than p_{co}—below the effective mobility edge—but large enough to avoid classical trapping. The evo-

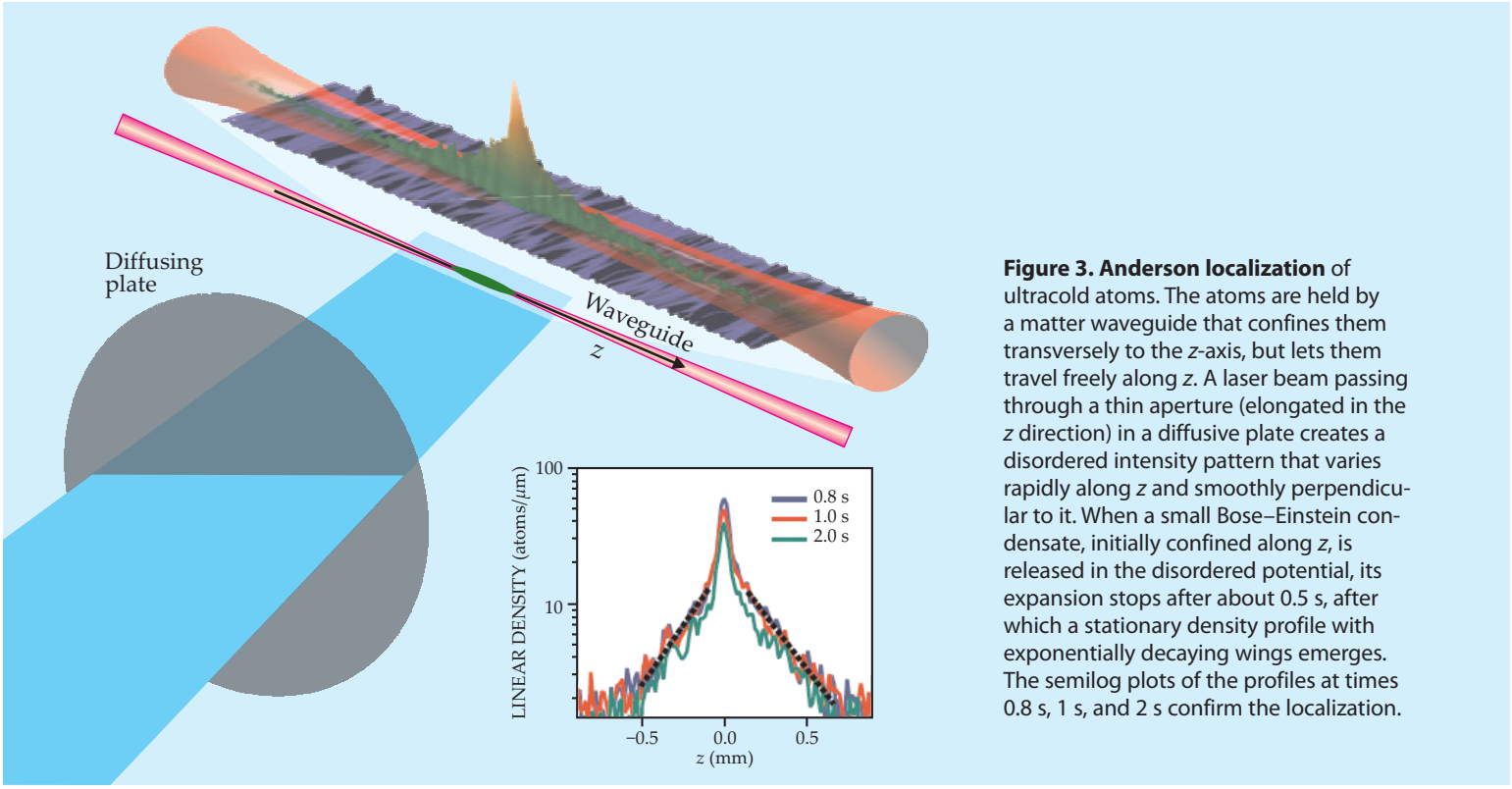


Figure 3. Anderson localization of ultracold atoms. The atoms are held by a matter waveguide that confines them transversely to the *z*-axis, but lets them travel freely along *z*. A laser beam passing through a thin aperture (elongated in the *z* direction) in a diffusive plate creates a disordered intensity pattern that varies rapidly along *z* and smoothly perpendicular to it. When a small Bose–Einstein condensate, initially confined along *z*, is released in the disordered potential, its expansion stops after about 0.5 s, after which a stationary density profile with exponentially decaying wings emerges. The semilog plots of the profiles at times 0.8 s, 1 s, and 2 s confirm the localization.

lution of the atomic wavefunction shows a striking behavior, as plotted in figure 3. After an initial expansion, the atomic wavepacket stops expanding and its wings decay exponentially. That is in stark contrast to the usual parabolic profile that never stops expanding, observed in the absence of disorder. We varied the intensity of the laser-induced speckle and checked that the localization length, or decay constant of the exponential profile, agrees with the prediction of the simplest theoretical approach sketched above, based on the Born approximation.

What happens beyond the effective mobility edge, when the wavefunction describing the expanding atoms has a momentum distribution with components beyond p_{co}? After some time only momentum components below p_{co} remain, and the result is a localized wavepacket. That wavepacket is localized with an algebraic rather than exponential profile, however.[8]

Actually, a more elaborate theoretical treatment based on an expansion beyond the Born approximation shows that some localization is still expected beyond the effective mobility edge,[12] but with a localization length orders of magnitude larger than our sample size. So although the general theorem that "there is always localization in one dimension" is not violated, strictly speaking, there is indeed in our situation an effective mobility edge—or more precisely, a crossover between two regimes.

The bichromatic lattice

At LENS we loaded the nearly noninteracting atoms into an artificial 1D "crystal of light" composed of equally spaced nodes of a laser standing wave. When the intensity of the optical lattice is strong enough, the system experimentally simulates the tight-binding model considered by Anderson in 1958. Atoms are trapped in the optical lattice sites (akin to electrons bound to a periodic arrangement of ions in a crystal) but can still hop from one site to a neighboring one with a tunneling energy J.

In the absence of disorder, the atoms can be described in terms of Bloch waves, quantum mechanical solutions for the motion of a particle in a periodic potential, and their energy spectrum shows the presence of allowed and forbidden bands. That band structure has been observed in a number of experiments in which ultracold atoms have been used to simulate the physics of electrons in ideal crystals. By adding disorder to the optical lattice, one can explore the transition from extended Bloch waves to exponentially localized states.

Figure 4a illustrates the experiment. A second optical lattice with different site spacing is added to the first. Superimposing the two periodic waves creates a potential without any periodicity, provided that the two wavelengths are not commensurate—that is, if their ratio is an irrational number. That condition breaks the discrete translational invariance of the main lattice, and the resulting lattice sites are energy-shifted from each another by quasi-random potential offsets.

The optical lattice introducing disorder is much weaker than the main lattice creating the tight-binding configuration. The weak lattice may thus be used to affect only the amplitude W of the energy offsets; the probability of hopping from one site to a nearest neighbor is basically constant across the lattice and is controlled by the height of the main lattice only. The amplitudes of the two lattices can thus be used to independently adjust both of the system's relevant energy scales.

This method for producing short-range inhomogeneities in the lattice has allowed us to explore interesting features of AL. In addition to simplicity, the method has two remarkable features: the possibility to continuously sweep between periodic and disordered systems by changing the degree of "irrationality" in the two lattice constants, and the existence of a metal–insulator transition in one dimension at a precise value of the ratio between the strength of disorder W and the tunneling energy J, akin to what happens in the Anderson model in higher dimensions.

Anderson localization in 1D bichromatic lattices is a well-known problem. Its theoretical treatment dates back 30 years ago with the work of Serge Aubry and Gilles André, who showed that a tight-binding model with a quasi-periodic disorder exhibits a phase transition when W is larger than $2J$.

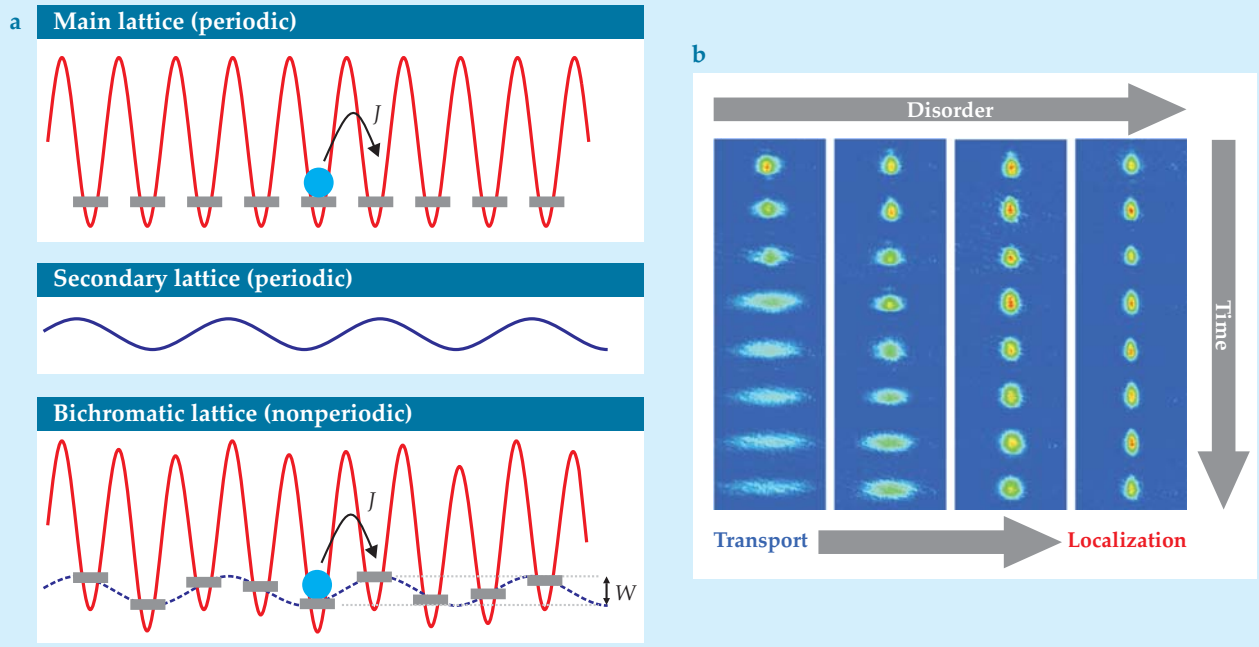


Figure 4. A bichromatic lattice simulates solid-state model. Loaded in a laser standing wave, an ultracold atom (blue) experiences a periodic potential. **(a)** The tunneling energy J can be controlled by changing the intensity of the main standing wave. A second, weaker optical lattice with an incommensurate spacing breaks the translational invariance and scrambles the site energies, as in the original solid-state model introduced by Philip Anderson. **(b)** Time-resolved images of almost noninteracting potassium-39 atoms. The atoms are first loaded into a few central sites of the bichromatic lattice and then observed diffusing into that nonperiodic structure. An increase in disorder leads to a decrease in diffusion and eventually its absence when the amount of disorder W becomes on the order of J.

Interactions among atoms

In a cold dilute atomic gas, atoms predominantly interact with each other through short-range van der Waals forces that can be described in terms of elastic s-wave collisions. Control over those collisions can be an interesting tool to investigate localization phenomena. In its original sense, AL is indeed a single-particle phenomenon, which demands the lack of interactions between the particles; repulsive interactions between the atoms can hamper localization.

The potassium-39 BEC is among those for which the strength of the collisions between the ultracold atoms can be conveniently tuned by using a static magnetic field.[14] The phenomenon, known as a Feshbach resonance, has led to many striking advances in cold atomic physics (see Daniel Kleppner's Reference Frame in PHYSICS TODAY, August 2004, page 12). Potassium-39 has a convenient resonance of that kind, which has been used to bring the strength of collisions between the BEC atoms to essentially zero. One can then look for localization induced entirely by disorder, with an ensemble of hundreds of thousands of noninteracting particles all occupying the same quantum state. It was possible to somewhat re-create the physical situation that Anderson considered in his article: putting one particle in a lattice site (or more precisely, 10^5 clones of the same particle initially occupying few sites) and studying the evolution in time. Figure 4b shows evidence of reduced expansion and exponential localization above a certain amount of disorder. The transition to complete freezing of the atomic motion occurs at a critical value of $W/J \approx 1$, no matter what tunneling time is used in the experiment, as predicted by Aubry and André.[15]

Observations at LENS have not been limited to transport phenomena. The localization transition was also studied by directly imaging the momentum distribution of the atoms as they passed from extended to localized states. If the optical lattices are suddenly switched off, the BEC wavefunction starts evolving in free space, and after a long

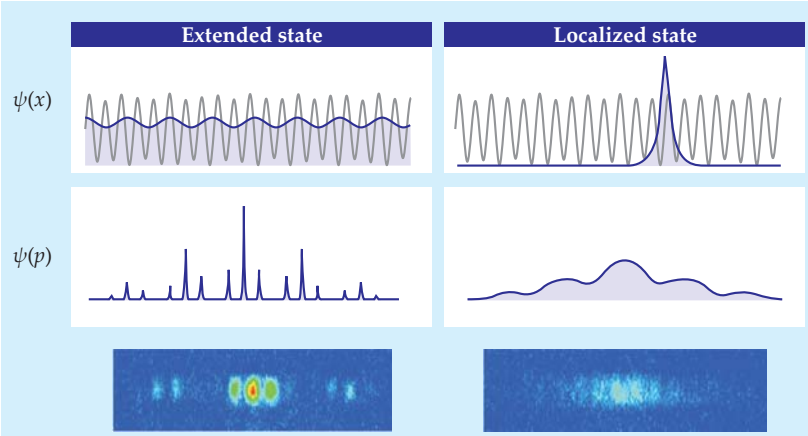


Figure 5. Imaging the phase transition in momentum space. When the atomic wavefunction is extended all over the bichromatic lattice, narrow peaks are recorded in the momentum distribution. When the atomic wavefunction is localized, a broad momentum spread is observed, as expected from the uncertainty principle. The false-color images in the bottom row are experimental maps of the momentum distribution recorded after a time-of-flight expansion in free space.

enough time, its squared modulus provides a map of the momentum distribution. A localized state should lead to a broad momentum distribution; an extended state is expected to yield a narrow distribution, as intuitively expected from the uncertainty relation. The measurement of the momentum distribution yields a quantitative description of the transition (see figure 5).

Toward quantum simulators

Even in cases as simple as 1D disordered potentials and noninteracting atoms, experimental studies have stimulated new theoretical approaches and led to unexpected findings such as the existence of an effective mobility edge in a speckle potential, which differs from the well-known case of a symmetric Gaussian-distributed disorder. When experimentalists go to higher dimensions, add interactions between the atoms, and can choose between fermion and boson atoms, they will be able to address a whole host of fascinating theoretical questions,[16] most of them bearing on many-body problems that raise formidable challenges to theorists.

A better understanding of the most general behavior of quantum particles in disordered potentials is important for applications as well. For instance, the interacting electrons in amorphous silicon used for photovoltaic cells are subject to a disordered potential. When the field matures enough that ultracold atom experiments yield quantitative results on such systems, then one can claim to have realized a quantum simulator, a device able to calculate more efficiently than available numerical techniques.[17]

Far from exhausting the subject, the recent experiments outlined here and elsewhere[18] prove the potential of ultracold atoms as a wonderful playground in which to study a problem that Anderson himself once thought out of reach. As he stated in his Nobel lecture,

> [A] reason why I felt discouraged in the early days was that I couldn't fathom how to reinsert interactions, and was afraid they, too, would delocalize. The realization that, of course, the Mott insulator localizes without randomness, because of interactions, was my liberation on this: one can see easily that the Mott and Anderson effects supplement, not destroy, each other. . . . The present excitement of the field for me is that I feel a theory of localization with interactions is beginning to appear. . . .

Thirty years later, such a theory is still wanting, but quantum simulators may tell us how nature behaves and provide theorists with clues about how to proceed.

The results reported here have been obtained by teams of theorists and experimentalists at Institut d'Optique (Juliette Billy, Vincent Josse, Zhanchun Zuo, Alain Bernard, Ben Hambrecht, Pierre Lugan, David Clément, Laurent Sanchez-Palencia, and Philippe Bouyer) and at the European Laboratory for Nonlinear Spectroscopy (Giacomo Roati, Chiara D'Errico, Leonardo Fallani, Marco Fattori, Chiara Fort, Matteo Zaccanti, Giovanni Modugno, and Michele Modugno). We acknowledge many fruitful discussions with Thierry Giamarchi, Maciej Lewenstein, Gora Shlyapnikov, Boris Altshuler, and others from whom we have learned a lot. After completing this manuscript, we were informed via private communication of similar results recently obtained in Randall Hulet's group at Rice University.

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